

BLADE DESIGN AND ANALYSIS USING A MODIFIED EULER SOLVER

N92-13952

O. Léonard and R. A. Van den Braembussche

von Karman Institute
Chaussée de Waterloo 72
1640 Rhode-Saint-Genèse
Belgium

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ABSTRACT

An iterative method for blade design based on an Euler solver and described in an earlier paper is used to design compressor and turbine blades providing shock free transonic flows. The method shows a rapid convergence, and indicates how much the flow is sensitive to small modifications of the blade geometry, that the classical iterative use of analysis methods might not be able to define.

The relationship between the required Mach number distribution and the resulting geometry is discussed. Examples show how geometrical constraints imposed upon the blade shape can be respected by using free geometrical parameters or by relaxing the required Mach number distribution.

The same code is used both for the design of the required geometry and for the off-design calculations. Examples illustrate the difficulty of designing blade shapes with optimal performance also outside of the design point.

SYMBOLS

a	speed of sound
M	isentropic Mach number
\vec{n}	normal vector
p^0	total pressure
p	static pressure
t	time
T^0	total temperature
\vec{V}	velocity vector
β	flow angle (with resp. axial)
σ	cascade solidity

subscripts

n	normal component
t	tangential component
1	cascade inlet
2	cascade outlet

INTRODUCTION

The design of new compressor and turbine blades is in most cases still done by successive direct analysis of the flow field around a given blade shape and modifications of the blade geometry, according to some empirical criteria and/or the designer's own experience. This approach makes it easier to respect geometrical and mechanical constraints imposed to the designer, such as thickness distribution, inertia momentum, stagger angle, pitch-to-chord ratio, etc.

New aerodynamic design tools have been developed, that have shown the ability to provide conclusive improvements of the aerodynamic performance when compared to existing blades. These improvements result from a specified controlled diffusion along the blade surface or a shock-free transonic flow. It is unlikely that they can be obtained by a traditional design procedure, namely by a series of flow analysis and empirical blade modifications. The design of transonic shock-free blades by means of an inverse method is one of the main topics discussed in this paper.

Analytical design methods developed in the past, using conformal mapping (Lighthill, 1945, Woods, 1955), permitted to build a complete theory of the inverse design of airfoils and blades, and provided the conditions required for the existence of a solution. However, they have a limited application due to the restrictive assumptions needed to allow an analytical formulation of the problem. As the blade shape results from the calculation, it is also more difficult to satisfy the mechanical constraints that one may wish to impose on the blade shape.

Numerical inverse methods have been developed for potential flows, using singularities for incompressible cases (Murugesan and Raily, 1969, Ubaldi, 1984, Van den Braembussche et al., 1989) and the hodograph plane (Bauer et al., 1972, Sanz, 1984) or the potential-stream function plane (Stanitz, 1953, Schmidt, 1980) for the compressible cases. The last methods are not very accurate in the stagnation point region and are unable to predict shocks. It is therefore questionable whether blades designed in this way for shock-free transonic flows are shock-free in reality.

Non potential flow fields require solving the Euler equations. Such methods are capable of treating shocks correctly and are therefore suited to verify shock-free designs. They are mostly used in iterative procedures and require a first guess of the blade shape. This initial geometry is modified from the results of a flow analysis until the imposed pressure or velocity distribution is reached. The blade modifications can be calculated in a pure mathematical way, in order to minimize an error function, eg. depending on the difference between the calculated pressure distribution and the target (Vanderplaats, 1979, Hicks, 1981). Although these methods have the capability to respect geometrical constraints, they are still very expensive in terms of CPU time, because many iterations and flow analyses are required.

The blade modification can be determined in a more physical way, resulting in decrease in CPU time. The present method imposes the required Mach number distribution as a boundary condition on the blade wall and uses the concept of a permeable wall to define the modification of the geometry. This approach allows a reduction of the number of blade modifications, and consequently of the number of mesh generations. The method has proven to be very efficient in subsonic and transonic applications (Léonard, 1990, Léonard and Van den Braembussche, 1991). As shown in this paper, the iterative procedure makes it easier to meet geometrical and mechanical constraints imposed in industrial applications, and to find out whether a realistic

blade shape corresponds to the required Mach number distribution. Another advantage of the present method is the possibility of using the same code for the blade design procedure as well as for the off-design analysis.

THE EULER SOLVER

The system of Euler equations for unsteady flows is solved using a time marching procedure and a finite volume approach. The numerical domain is discretized using C grids, for a good description of the leading edge geometry (fig.1). The unknowns are located at the vertices of the mesh cells, in such a way to avoid extrapolation towards the blade wall. The code can handle open trailing edges, in order to allow additional degrees of freedom in the geometrical definition of the blades. This makes the problem of solution existence easier to solve and allows a sufficient blade thickness to contain the boundary layer.

The equations are integrated in time using a Runge-Kutta first order accurate scheme, with local time-stepping, enthalpy damping and implicit residual averaging to accelerate the convergence. A detailed description of the solver may be found in Léonard (1990).

CALCULATION OF THE UNKNOWN ON THE BLADE WALL

The method developed by the authors is an "iterative inverse method", in which the final geometry is the result of the flow calculation, imposing the required Mach number distribution on the blade wall. It has to be iterative since the location of this boundary is part of the solution, approximated at the beginning of the design procedure by any convenient initial geometry. There is no reason that the flow remains tangent to this geometry during the calculation, except in two cases, when the blade "is" the solution of the problem or when the blade wall is modified in order to respect the slip condition, as the time marching procedure iterates to the steady state.

Methods based on the second case have been proposed by Meauzé (1982), Giles and Drela (1987), and Zannetti et al. (1984). This approach has not been considered here since a minimum of successive blade modifications and corresponding mesh generations is desired. The blade wall must therefore be treated as permeable to the flow field. After convergence of the time marching procedure, the flow calculation results in a distribution of a normal velocity component on the blade wall that is used to modify the geometry.

The calculation of the unknowns at a boundary is dominated by the mathematical nature and the physical properties of the system of equations. As the Euler unsteady equations are hyperbolic, the solution can be constructed, at any location in the calculation domain (including the boundaries) using the information propagating in directions perpendicular to characteristic surfaces. The eigenvalues of the Jacobian matrices of the Euler system, projected in a considered direction \vec{n} , are $\vec{V}\vec{n}$, $\vec{V}\vec{n}$, $\vec{V}\vec{n} + a$ and $\vec{V}\vec{n} - a$, and define the propagation speeds in that direction. If the vector \vec{n} is chosen perpendicular and entering the blade wall, a positive speed means that the information is propagated on the wave front, in the \vec{n} direction, from the inside of the calculation domain to the outside, and is therefore available to calculate the value of the unknowns at this point of the blade wall. On the other hand, a negative speed means that the information comes from the outside of the numerical domain and propagates towards the inside. This entering information has to be provided by a boundary condition at the boundary point.

If the slip condition is imposed on the blade wall ($\bar{V}\bar{n} = 0$) only the speed $\bar{V}\bar{n} - a$ is negative and therefore only one boundary condition must be imposed, i.e. the velocity direction at that point. This shows that the slip condition can not be imposed together with the Mach number value, at least for a fixed blade wall.

On the other hand, if the static pressure p (or the Mach number) is imposed on the blade, a velocity component normal to the blade can appear and, depending on its sign, 1 or 3 eigenvalues will be negative and 0 or 2 additional conditions must be imposed. The sign of this normal velocity component can be determined as a function of the imposed static pressure, using the compatibility relation corresponding to the only eigenvalue which is always positive ($V_n + a$).

If the normal velocity is positive, one boundary condition (the required static pressure) must be imposed, since only one eigenvalue ($V_n - a$) is negative. The additional information necessary to calculate all the unknowns at the boundary can be provided by the two compatibility relations corresponding to V_n and V_n since they are positive.

If the normal velocity is negative, two additional boundary conditions must be imposed. The best results have been obtained by imposing the total pressure and total temperature at that point. Imposing the latter does not give any special problem, since in a blade-to-blade calculation it is supposed to remain equal to the total temperature at the inlet. Imposing the total pressure is not so straightforward because of numerical dissipation. This problem is solved by imposing the value of the total pressure from the previous time level in such a way that the total pressure can adapt to the new flow field. This is important when a shock-free design is performed starting from a blade for which a shock was present in the original flow field, since in this case the initial and final total pressure distributions on the blade wall can be very different from each other. A detailed derivation of the compatibility relations can be found in Léonard (1990).

MODIFICATION OF THE GEOMETRY

A new geometry must be found since the initial shape no longer corresponds to a streamline. The modification algorithm is based on a transpiration model and calculates the position of the new streamlines using the velocity component normal to the initial blade (Léonard, 1990). The modification starts at the stagnation point, and is performed separately for the pressure side and the suction side. The new suction and pressure sides are defined as streamlines of the flow satisfying the Euler equations, and therefore can not cross each other. This guarantees a blade with positive thickness if the numerical integration procedure and the normal velocity calculation are sufficiently accurate.

RESULTS

The first example illustrates the accuracy of the method for shock-free transonic flows by applying it to the supercritical compressor blade designed by Sanz (1984) with an odograph method, and proposed as a test case for inviscid calculation methods in AGARD-AR-275 (fig. 2a). Analysis of the flow with the present method shows discrepancies on the suction side Mach number distribution (fig. 2b) similar to the ones observed by Denton (1983).

The geometry calculated by Sanz has been redesigned using the present method in order

to obtain the shock-free Mach number distribution imposed by Sanz as the input data of his design (fig. 2c). Only one modification of the geometry has been necessary to obtain good agreement (fig. 2d). The difference between the initial geometry designed by Sanz and the one designed with the present method is very small. This example suggests that the original geometry defined by Sanz may not be shock-free, and illustrates how supersonic flows are very sensitive to geometry changes.

A second example illustrates the design of a shock-free compressor blade, using a NACA-65 (12A₂I_{8b})10 as an initial geometry. This blade is not suited to transonic flows, and a strong shock is present in the flow field. Therefore large geometry modifications are expected. The flow conditions are: $M_1 \simeq 0.8$, $p_1^0 = 1.33$ bar, $T_1^0 = 341.5$ K, $\beta_1 = 45$ deg, $M_2 = 0.5$. The cascade geometry is defined by a stagger angle of 31 deg and a solidity of 1.

In a first design, only the suction side Mach number distribution has been modified. The initial distribution is compared to the shock-free required distribution in figure 3a. Good agreement between the calculated and the imposed Mach number distributions is obtained after 4 blade modifications (fig. 3b). The final blade is compared to the NACA-65 blade in figure 3c. One observes a thick leading edge, due to the velocity peak in the pressure side leading edge region. This is not desirable because it leads to strong diffusion and subsequent flow separation along the pressure side, as predicted by a boundary layer calculation.

A second design has been performed, starting also from the NACA-65 blade, but by modifying both the pressure and suction side Mach number distributions (fig. 4a). Decreasing the pressure side velocity in the leading edge region results in a lower average velocity, and in a smaller leading edge thickness because continuity requires a smaller blade blockage. Convergence to the required distribution is obtained after 3 modifications of the geometry (fig. 4b). The initial and final geometries are compared in figure 4c. One can observe a thinner leading edge and a shift of the maximum thickness location towards the middle of the blade.

Blade shapes designed by inviscid methods include the boundary layer blockage on the pressure and suction sides. The physical blade geometry can be obtained by subtracting the boundary layer displacement thickness from the so-called "inviscid" geometry. The minimum thickness of the "inviscid" blade, required to contain the boundary layer, can be calculated as a function of the target velocity distribution before the design procedure is started.

The analysis of the boundary layer for the prescribed Mach number distribution shown on figure 4b indicates that the boundary layer thickness at the trailing edge is of the order of 5% of the chord length, which is larger than the total trailing edge thickness of the blade shown on figure 4c and makes this blade unphysical. Increasing the trailing edge thickness is possible by increasing both the suction and pressure side Mach number distributions in the trailing edge region by the same amount (fig. 5a). The circulation is unchanged, resulting in the same turning of the flow, but the blade thickness must increase to satisfy continuity. The redesigned geometry is compared to the previous design in figure 5b and shows a larger trailing edge thickness capable of enclosing the boundary layer and the mechanical thickness.

An off-design analysis of the second blade has been performed with the same solver, changing the incidence by ± 2 degrees (fig. 6a and 6b). One can observe that the shock reappears. Although the flow field is no longer shock-free, the off-design behaviour of the new blade is better than that of the initial geometry.

The third example illustrates the redesign of a transonic turbine blade. The starting geometry is taken from the workshop VKI-LS 82-05 (Arts, 1982). The flow conditions are: $p_1^0 = 1$ bar, $T_1^0 = 278$ K, $\beta_1 = 0$ deg, $M_2 = 1.1$. The cascade geometry is defined by a stagger angle of -60 deg and a solidity of 1.25.

The imposed shock-free Mach number distribution assures a monotonically increasing velocity on the suction side (fig. 7a). Two modifications of the blade geometry are sufficient to give good agreement between the calculated and the required Mach number distributions (fig. 7b). The original and final geometries are compared in figure (7c). Off-design distributions are shown in figures 7d for an exit Mach number of 1.05 and 1.15 instead of 1.1.

The number of grid nodes used in the above examples ranges from 161×15 to 199×15 . The typical amount of CPU time for one blade modification is 15 minutes on an ALLIANT FX/8 computer with 5 processors.

CONCLUSIONS

The present method has been successfully used to design shock-free transonic blades. It provides in few iterations results that could not be achieved using traditional direct methods and empirical blade modifications.

The method combines the advantages of a pure inverse method, since the Mach number distribution can be imposed on the blade wall, and the advantages of a direct method, allowing good control of the geometrical parameters.

It has been shown how modifications of the required Mach number distribution influence the blade geometry. Special attention was given to design trailing edges of sufficient thickness to enclose the boundary layer blockage.

Off design analysis of designed geometries illustrate the difficulty of optimizing for more than one operating point.

REFERENCES

- ARTS, T., 1982, "Numerical methods for flow in turbomachine bladings", *Workshop on 2D and 3D flow calculations in turbine bladings*, VKI LS 1982-05.
- BAUER, F., GARABEDIAN, P. and KORN, D., 1972, *Supercritical Wing Sections*, Vol. I, New York: Springer-Verlag.
- DENTON, J.D., 1983, "An Improved Time-Marching Method for Turbomachinery Flow Calculation", *ASME Journal of Engineering for Power*, Vol. 105, No. 3, pp 514-524.
- GILES, M.B. and DRELA, M., 1987, "Two-Dimensional Transonic Aerodynamic Design Method", *AIAA Journal*, Vol. 25, No. 9, pp 1199-1206.
- HICKS, R.M., 1981, "Transonic Wing Design Using Potential Flow Codes - Successes and Failures", SAE Paper 810565.
- JAMESON, A., SCHMIDT, W. and TURKEL, E., 1981, "Numerical Solutions of the Euler Equations by Finite Volume Methods Using Runge-Kutta Time Stepping Schemes", AIAA Paper 81-1259.
- LEONARD, O., 1990, "Subsonic and Transonic Cascade Design", AGARD-R-780.
- LEONARD, O., and VAN DEN BRAEMBUSSCHE, R., 1991, "Design Method for Subsonic and Transonic Cascade with Prescribed Mach Number Distribution", ASME Paper 91-GT-18.

- LIGHTHILL, J.M., 1945, "A New Method of Two-Dimensional Aerodynamic Design", ARC R&M 2112.
- MEAUZE, G., 1982, "An Inverse Time Marching Method for the Definition of Cascade Geometry", *Journal of Engineering for Power*, Vol. 104, pp 650-656.
- MURUGESAN, K. and RAILLY, J.W., 1969, "Pure Design Method for Airfoils in Cascade", *Journal Mechanical Engineering Science*, Vol. 11, No. 5, pp 454-465.
- SANZ, J.M., 1984, "Improved Design of Subcritical and Supercritical Cascades Using Complex Characteristics and Boundary-Layer Correction", *AIAA Journal*, Vol. 22, No. 7.
- SCHMIDT, E., 1980, "Computation of Supercritical Compressor and Turbine Cascades with a Design Method for Transonic Flows", *Journal of Engineering and Power*, Vol. 102, pp 68-74.
- STANITZ, J.D., 1953, "Design of Two-Dimensional Channels with Prescribed Velocity Distributions along the Channel Walls", NACA R-1115.
- UBALDI, M., 1984, "Un metodo di progetto per profili in schiera basato sulla teoria delle equazioni integrali", AIMETA, VII Congr. Nazionale, Trieste.
- VAN DEN BRAEMBUSSCHE, R., LEONARD, O. and NEKMOUCHE, L., 1989, "Subsonic and Transonic Blade Design By Means of Analysis Codes", AGARD-CP-463.
- VANDERPLAATS, G.N., 1979, "Approximation Concepts for Numerical Airfoils Optimization", NASA TP-1370.
- ZANNETTI, L. and PANDOLFI, M., 1984, "Inverse Design Technique for Cascades", NASA CR-3836.
- WOODS, L.C., 1955, "The design of two-dimensional airfoils with mixed boundary conditions", *Quarterly of Applied Mathematics*, Vol. 13, pp. 139-146.

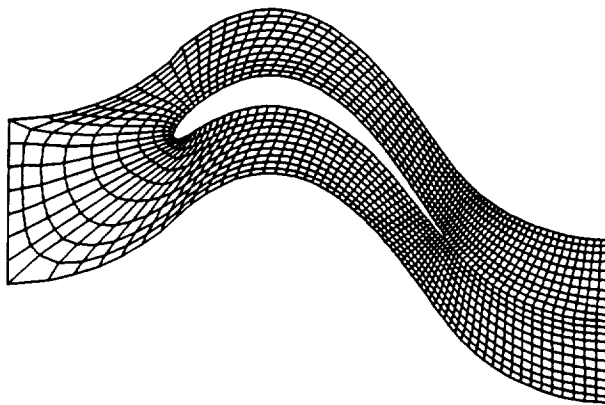


Fig. 1 C-grid discretization for a turbine blade

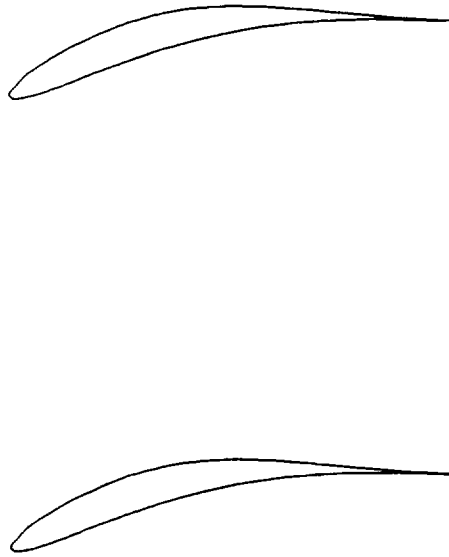


Fig. 2a Sanz supercritical compressor blade

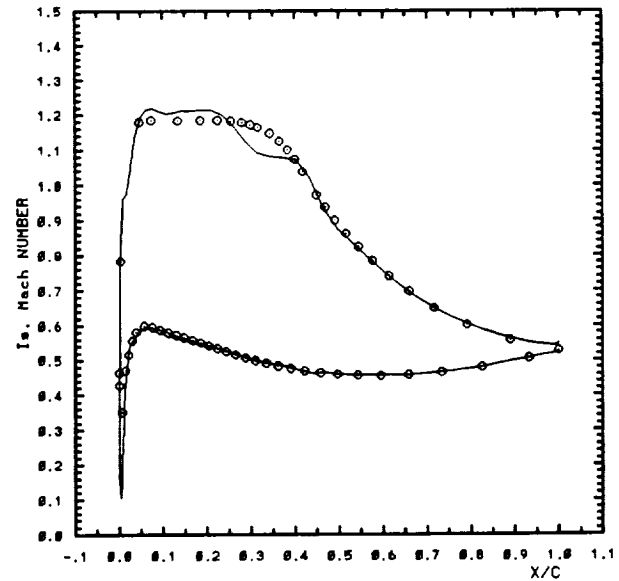


Fig. 2b Analytical (○) and calculated (full line) Mach number distributions

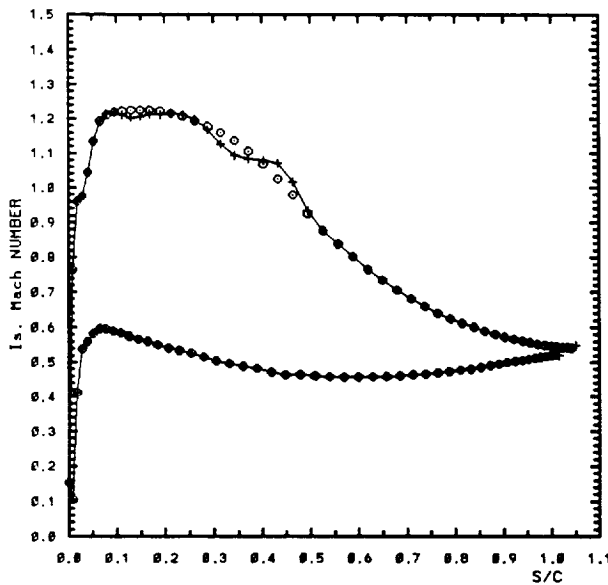


Fig. 2c Initial (+) and required (○) Mach number distributions

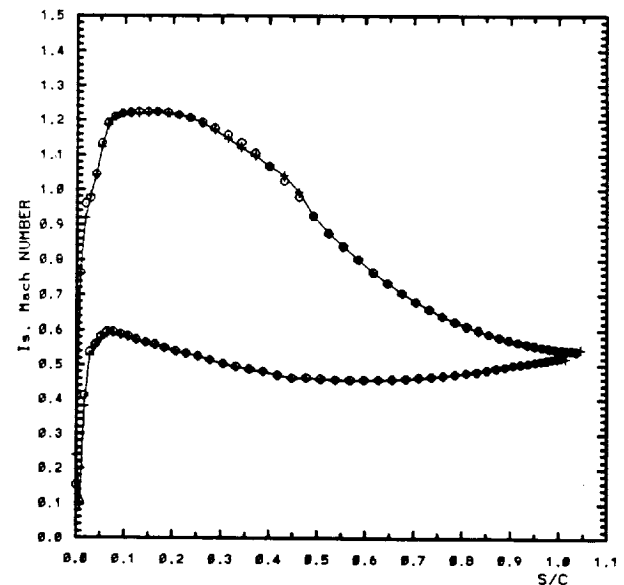


Fig. 2d Calculated (+) and required (○) Mach number distributions after 1 modification

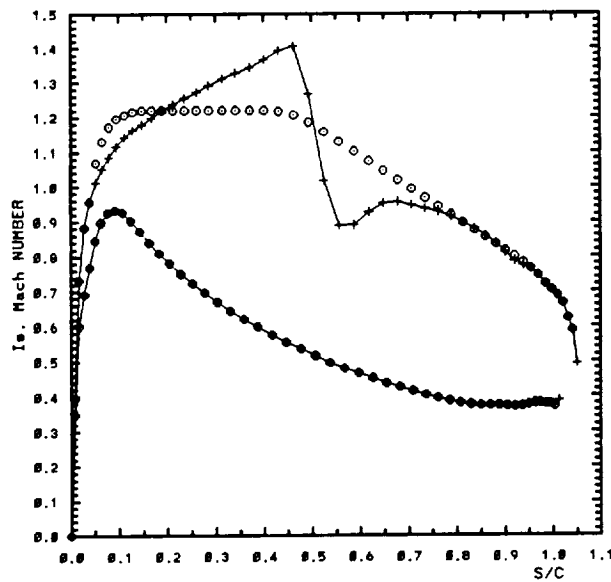


Fig. 3a Initial (+) and required (O) Mach number distributions

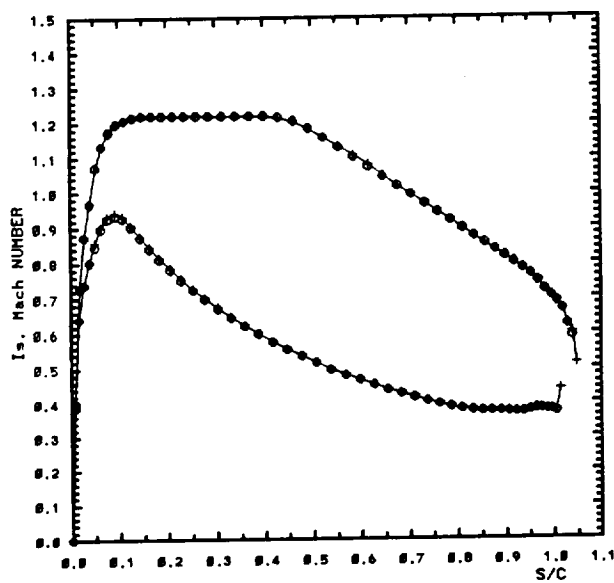


Fig. 3b Calculated (+) and required (O) Mach number distributions after 3 modifications

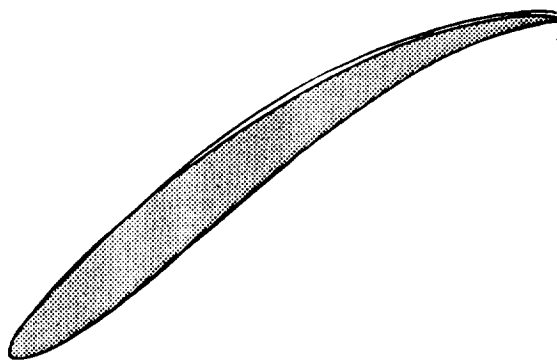


Fig. 3c Original and final geometries

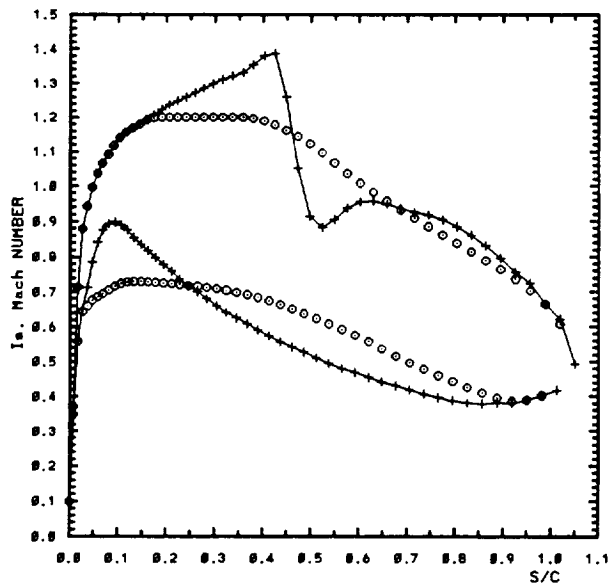


Fig. 4a Initial (+) and required (O) Mach number distributions

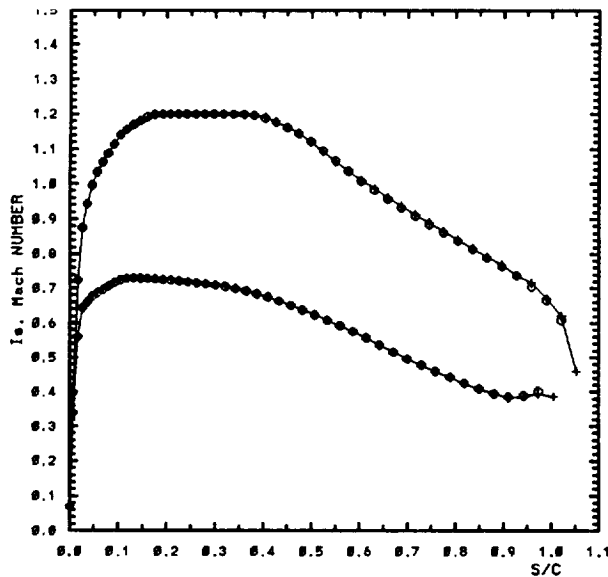


Fig. 4b Calculated (+) and required (O) Mach number distributions after 3 modifications

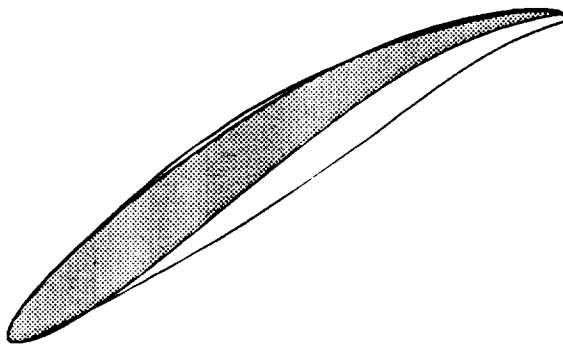


Fig. 4c Original and final geometries

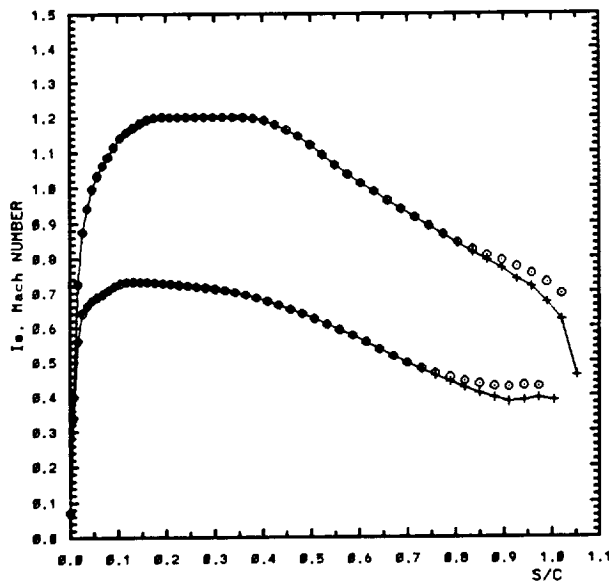


Fig. 5a Initial (+) and required (○)
Mach number distributions for
increased blade thickness

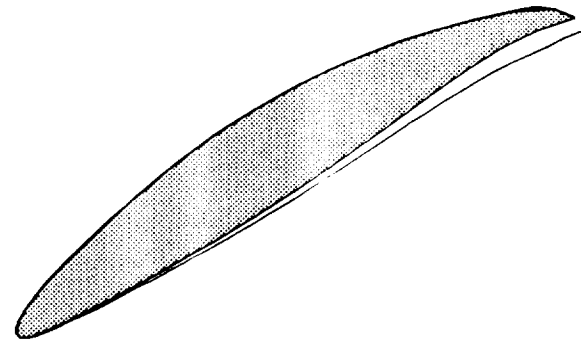


Fig. 5b Original and thicker
geometries

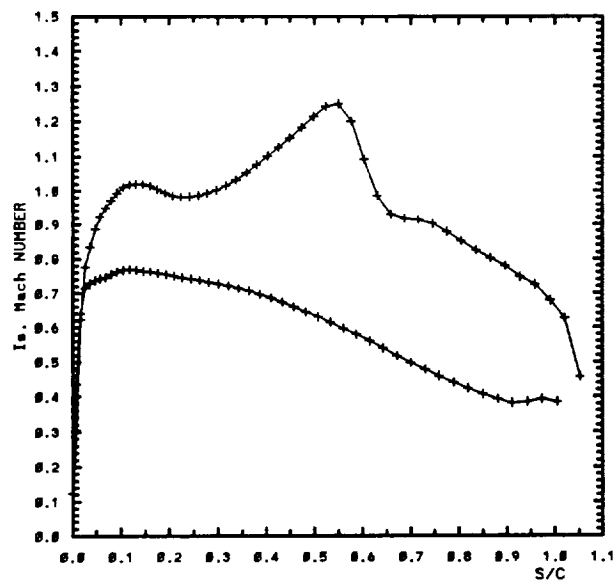


Fig. 6a Off design
(-2 deg incidence)

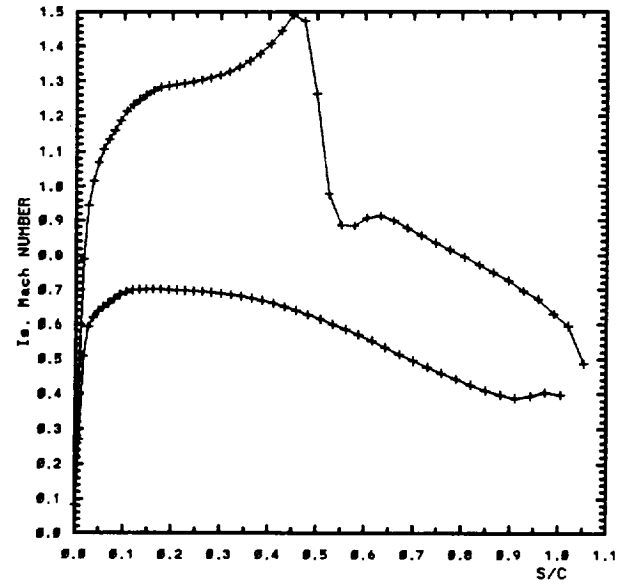


Fig. 6b Off design
(+2 deg incidence)

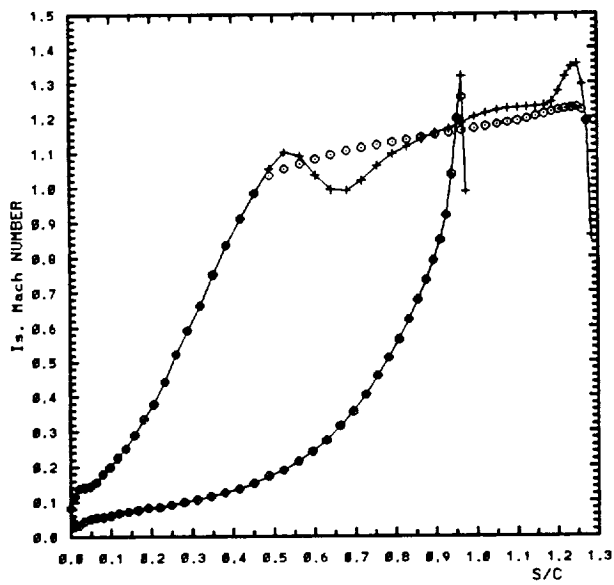


Fig. 7a Initial (+) and required (○) Mach number distributions

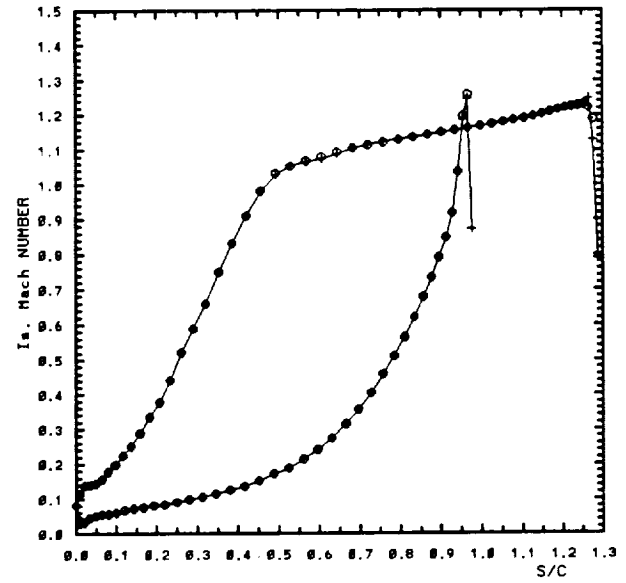


Fig. 7b Calculated (+) and required (○) Mach number distributions after 2 modifications

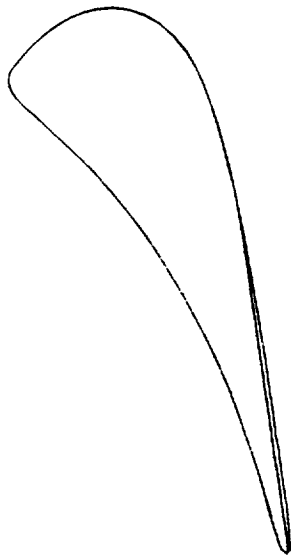


Fig. 7c Original and final geometries

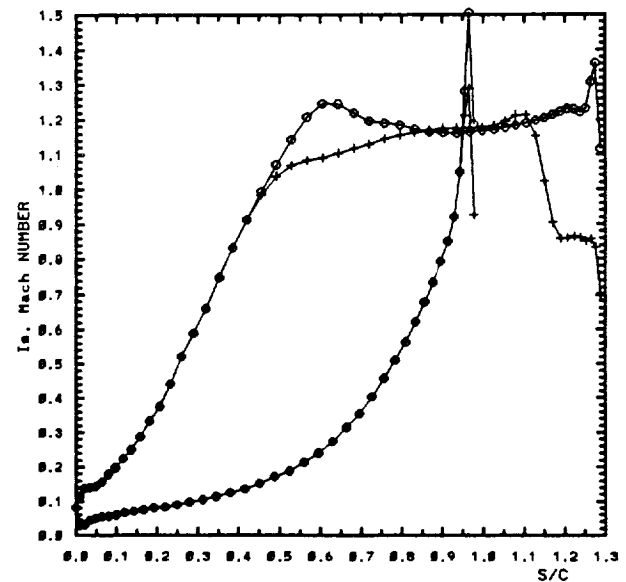


Fig. 7d Off design for $M_2 = 1.05$ (+) and $M_2 = 1.15$ (○)